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**Code No. : 6372** Sub. Code : ZMAM 24

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Second Semester

Mathematics – Core

DIFFERENTIAL GEOMETRY

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. The arc length from a to any point $u$ is given by

   (a) $r = S(u) = \int_a^u R(u)du$

   (b) $r = S(u) = \int_a^u |\dot{R}(u)|du$

   (c) $r = S(u) = \int_a^u |R(u)|du$

   (d) $r = S(u) = \int_a^u |\dot{R}(u)|^2 du$

2. The line of intersection of the nominal plant and the osculating plane at $P$ is called

   (a) the principal normal

   (b) the principal tangent

   (c) the principal curvature

   (d) the principal line

3. The radius of spherical curvature is given by

   (a) $R = \sqrt{\rho^2 + \sigma^2\rho^2}$ (b) $R = \sqrt{\rho^2 + \rho'^2}$

   (c) $R = \sqrt{\rho^2 + \sigma^2\rho'^2}$ (d) $R = \sqrt{\rho^2 + \sigma^2}$

4. The equation of the involute is

   (a) $R = r + t$ (b) $R = r + \rho n + \sigma\rho' b$

   (c) $[R - r, \dot{r}, \ddot{r}] = 0$ (d) $R = r + (c - s)t$

5. For the paraboloid $x = u$, $y = v$, $z = u^2 - v^2$, $F$ is

   (a) $1 + 4u^2$ (b) $1 + 4v^2$

   (c) $4uv$ (d) $-4uv$

6. The two parametric curves through a point $P$ are orthogonal if at $P$

   (a) $r_1 \times r_2 = 0$ (b) $r_1 \cdot r_2 = 0$

   (c) $r_1 + r_2 = 0$ (d) $r_1 \times r_2 \neq 0$

7. When $v = c$ for all values of $u$, a necessary and sufficient condition that the curve $v = c$ is a geodesic is

(a) $EE_2 + FE_1 + 2EF_1 = 0$

(b) $EE_2 + FE_1 - 2EF_1 = 0$

(c) $GC_1 + FG_2 - 2GF_2 = 0$

(d) $EE_2 - FE_1 + 2EF_1 = 0$

8. A necessary and sufficient condition for a curve $u = u(t)$, $v = v(t)$ on a surface $r = r(u, v)$ to be geodesic is that

(a) $U\frac{\partial T}{\partial \dot{v}} + V\frac{\partial T}{\partial \dot{u}} = 0$

(b) $U\frac{\partial T}{\partial v} - V\frac{\partial T}{\partial u} = 0$

(c) $U\frac{\partial T}{\partial \dot{v}} - V\frac{\partial T}{\partial \dot{u}} = 0$

(d) $U - V\frac{\partial T}{\partial \dot{u}} = 0$

9. The second fundamental form is

(a) $Edu^2 + 2Fdudv + Gdv^2$

(b) $Ldu^2 + 2Mdudv + Ndv^2$

(c) $Lh^2 + Mh\,k + N$

(d) $Ldv^2 + 2Mdu + Ndu^2$



10. The Gaussian Curvature $K$ is defined by

(a) $K = \frac{1}{2}(K_a + K_b)$ (b) $K = K_a K_b$

(c) $K = \frac{1}{2}\sqrt{K_a K_b}$ (d) $K = LN - M^2$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $[r', r'', r'''] = \kappa^2 \tau$.

Or

(b) Find the curvature and the torsion of the curve given by $r = \{a(3u - u^3), 3au^2, a(3u + u^3)\}$.

12. (a) Prove that the osculating plane at any point $P$ has three point contact with the curve at $P$.

Or

(b) Prove that the projection $C_1$ of a general helix $C$ on a plane perpendicular to its axis has its principal normal parallel to the corresponding principal normal of the helix and its corresponding curvature is given be $k = k_1 \sin^2 \alpha$.

13. (a) Show that the matrix is a positive definite quadratic form in $du, dv$.

Or

(b) Find E,F,G and H for the paraboloid

$x = y, y = v, z = u^2 - v^2$

14. (a) Prove that, on the general surface, a necessary and sufficient condition that the curve $v = c$ be a geodesic is $EE_2 + FE_1 - 2EF_1 = 0$ when $v = c$ for all values of $u$.

Or

(b) On the paraboloid $x^2 - y^2 = z$ find the orthogonal trajectories of the sections by the planes $z$ = constant.

15. (a) Prove that if the orthogonal trajectories of the curves $v$ = constant are geodesics, then $H^2 / E$ is independent of $u$.

Or

(b) If K is the normal curvature in a direction making an angle $\psi$ write the principle direction $v$ = constant then prove that $K = K_a \cos^2 \psi + K_b \sin^2 \psi$.



PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Serret Frenet formulae.

Or

(b) Show that the length of the common perpendicular $d$ of the tangents at two near points distance $s$ apart is approximately given by $d = \frac{k \tau s^3}{12}$.

17. (a) Define the osculating sphere and the centre A spherical curvature. If a curve lies on a sphere show that $\rho$ and $\sigma$ are related by $\frac{d}{ds}(\sigma\rho') + \frac{\rho}{\sigma} = 0$.

Or

(b) Prove that $\frac{K}{\tau}$ = constant is a characteristic property of helices.

18. (a) Find $E$, $F$, $G$ and $H$ for the anchor ring and find the area of the anchor ring corresponding to the domain $0 \le u \le 2\pi$ and $0 \le v \le 2\pi$.

Or

(b) If $(l', m')$ are the direction coefficients of a line which makes an angle $\pi/2$ with the line whose direction coefficients are $(l, m)$, then prove that $l' = -\frac{1}{H}(Fl + Gm), m' = \frac{1}{H}(El + Fm)$.

19. (a) Prove that every helix on an cylinder is a geodesic.

Or

(b) Prove that any curve $u = u(t), v = v(t)$ on a surface $r = r(u, v)$ is a geodesic if and only if the principal normal at every point on the curve is normal to the surface.

20. (a) State and prove Liouville's formula for Kg.

Or

(b) Prove that a necessary and sufficient condition for a curve on a surface to be a line of curvature is $kdr + dN = 0$ at each point on the line of curvature where K is the normal curvature in the direction $dr$ of the line of curvature.